AD-778 956

# COUNTEREVASION STUDIES

Stephen S. Lane

Texas Instruments, Incorporated

Prepared for:

Air Force Office of Scientific Research
Advanced Research Projects Agency

15 November 1973

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UNCLASSIFIED
Security Classification

AD-778956

**DOCUMENT CONTROL DATA - R & D**

*(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)*

| | |
|---|---|
| 1. ORIGINATING ACTIVITY (Corporate author)<br>Texas Instruments Incorporated<br>Equipment Group<br>Dallas, Texas 75222 | 2a. REPORT SECURITY CLASSIFICATION<br>UNCLASSIFIED<br>2b. GROUP |

3. REPORT TITLE
Counterevasion Studies

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)
Semi-Annual Technical Report No.1-Part B 1 May 1973 - 31 October 1973

5. AUTHOR(S) (First name, middle initial, last name)
Stephen S. Lane

| 6. REPORT DATE | 7a. TOTAL NO. OF PAGES | 7b. NO. OF REFS |
|---|---|---|
| 15 November 1973 | 46 | 4 |

| | |
|---|---|
| 8a. CONTRACT OR GRANT NO.<br>Contract No. F44620-73-C-0055<br>b. PROJECT NO.<br>c. ARPA Program Code No. F10<br>d. | 9a. ORIGINATOR'S REPORT NUMBER(S)<br>ALEX(02)-TR-73-01-PART B<br>9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)<br>AFOSR - TR - 74 - 0745 |

10. DISTRIBUTION STATEMENT

| | |
|---|---|
| 11. SUPPLEMENTARY NOTES<br>ARPA Order No. 1827 | 12. SPONSORING MILITARY ACTIVITY<br>Advanced Research Projects Agency<br>Nuclear Monitoring Research Office<br>Arlington, Virginia 22209 |

13. ABSTRACT


This report discusses progress to date on Contract No. F44620-72-C-0055. The complex cepstrum and eigenspectrum techniques are examined, and limits of their resolution for realistic synthetic data obtained.




DD FORM 1 NOV 65 1473



UNCLASSIFIED
Security Classification

| 14. KEY WORDS | LINK A | | LINK B | | LINK C | |
|---|---|---|---|---|---|---|
| | ROLE | WT | ROLE | WT | ROLE | WT |
| Interfering Events | | | | | | |
| Counterevasion | | | | | | |
| Eigenspectrum | | | | | | |
| Complex Cepstrum | | | | | | |

# COUNTEREVASION STUDIES

## SEMI-ANNUAL TECHNICAL REPORT NO. 1 - PART B
## 1 MAY 1973 TO 31 OCTOBER 1973

Prepared by
Stephen S. Lane

TEXAS INSTRUMENTS INCORPORATED
Equipment Group
Post Office Box 6015
Dallas, Texas 75222

Contract No. F44620-73-C-0055
Amount of Contract: $148,244
Beginning 23 April 1973
Ending 31 December 1973

Prepared for
AIR FORCE OFFICE OF SCIENTIFIC RESEARCH

Sponsored by
ADVANCED RESEARCH PROJECTS AGENCY
Nuclear Monitoring Research Office
ARPA Program Code No. F10
ARPA Order No. 1827

15 November 1973

Acknowledgment: This research was supported by the Advanced Research Projects Agency, Nuclear Monitoring Research Office, under Project VELA-UNIFORM, and accomplished under the direction of the Air Force Office of Scientific Research under Contract F44620-73-C-0055.

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## SECTION I
## INTRODUCTION

This report summarizes results to date on a study to determine the effectiveness of two methods which may be used to separate interfering seismic events. These techniques have potential value for counterevasion studies, in that they are expected to distinguish underground nuclear explosions hidden in larger earthquakes, and to identify sequential underground nuclear explosions disguised as earthquakes.

The techniques are applied to markedly different combinations of interfering waveforms. The complex cepstrum technique separates signals from the same azimuth, whose waveforms and thus spectra are similar. It does so by treating the entire spectrum of the data. The eigenspectrum technique, by contrast, is used to separate signals from different azimuths, with greatly different spectra, and considers the data over a narrow frequency band.

The complex cepstrum technique is discussed in great detail by Schafer (1969), and applied by him to removal of echos in speech. Booker and Ong (1972) discuss the eigenspectrum technique as applied to synthetic data. Section II of this report discusses the cepstrum technique as applied to seismic data, and presents results on artificially constructed interfering signals. An analysis of a possible genuine multiple event is also given. Section III develops the theory of the eigenspectrum and discusses results obtained with this technique as applied to interfering chirp waveforms in the presence of noise.

## SECTION II

## COMPLEX CEPSTRUM TECHNIQUE

### A. INTRODUCTION

The complex cepstrum is a technique used to separate a function consisting of the convolution of an oscillatory function with a series of sharp peaks. In the seismic case, the oscillatory function is a waveform observed at teleseismic distances from an earthquake or presumed underground nuclear explosion. The sharply peaked function, the multipath operator, represents either the multipathing nature of the earth, or the fact that more than one seismic event contributed to the observed waveform.

If the multipath operator m(t) consists of a single delta function at time zero, its convolution with the seismic waveform returns just the original waveform. If m consists of a number of delta functions at times 0, $T_1, \ldots, T_n$, with amplitudes 1, $a_1, \ldots, a_n$ the convolution yields the original waveform plus the waveform multiplied by $a_1$ and delayed by $T_1$, plus the waveform multiplied by $a_2$ and delayed by $T_2$, and so on.

Such a multipath operator might come about if the seismic disturbance generated by the event traveled to the observation point by a number of paths, each of which attenuated the motion differently. For surface waves, continental margins or mountain chains can cause multipathing. Horizontal reflection planes deep in the earth can cause multiple arrivals for bodywaves.

Another cause of multiple arrivals is a multiple source. Earthquakes are sometimes followed by aftershocks at quite short time intervals, and underground nuclear detonations may consist of a number of events separated in time, for a variety of reasons. Waves from such events need only follow one path to be observed as multiple arrivals.

The assumption that the multipath operator consists of a series of delta functions is the same as the assumption that there is no phase distortion between the arrivals, and that they differ in amplitude but not in shape. This will be the case when the multiple paths followed have the same attenuation and dispersion as a function of frequency, or when the source mechanisms of the contributing events are the same. Obviously these conditions are never met exactly, and the multipath operator will not be a series of delta functions. Satisfactory results may still be achieved, however, if the later arrivals are not too distorted.

## B. DESCRIPTION OF THE TECHNIQUE

We now put the above in mathematical terms. This has been done in a rigorous fashion by Shafer (1969) and Ulrych (1971). The present treatment is intended to stress the physical significance of the various steps.

### 1. Modulation of the Spectrum

Suppose that in the simplest case the observed waveform is given by:

$$X(t) = S(t) + a\,S(t-T) \tag{1}$$

consisting of a first arrival $S(t)$ and a secondary arrival delayed by $T$ and scaled by $a$. The second arrival is a duplicate of the first in shape. The total signal can be written as a convolution:

$$X(t) = S(t) * m(t) \tag{2}$$

where the multipath operator is $m(t) = \delta(t) + a\,\delta(t-T)$

Then the Fourier transform of the total signal is:

$$\tilde{X}(f) = \tilde{S}(f)\left(1 + ae^{i2\pi fT}\right) \qquad (3)$$

Writing the spectrum in polar form:

$$\tilde{X}(f) = |S(f)|\, e^{i\psi(f)} \sqrt{1+a^2+2a\cos 2\pi fT}\; e^{i\phi(f)} \qquad (4)$$

where $\phi(f) = \tan^{-1} \dfrac{a \sin 2\pi fT}{1+a\cos 2\pi fT}$

and $\psi(f)$ is the phase spectrum of the unmultipathed signal.

Consider first the amplitude of $\tilde{X}(f)$. If the spectrum of the signal alone varies smoothly, the effect of multiplying it by $\sqrt{1+a^2+2a\cos 2\pi fT}$ is to produce regular minima at a frequency spacing of $\Delta f = 1/T$. For the case where $a = 1$ the amplitude at the minima is zero. For all other values of a the amplitude there is greater than zero. We may say that the amplitude spectrum is modulated by the multipath operator.

The logarithm of the amplitude is taken before further processing. The log amplitude spectrum is then:

$$\ln|\tilde{S}(f)| + \frac{1}{2} \ln(1+a^2+2a\cos 2\pi fT) \qquad (5)$$

The modulation is now additive rather than multiplicative, and has constant amplitude across the frequency band.

Next consider the phase spectrum for the special case where $a = 1$. Then:

$$\varphi = \tan^{-1} \frac{\sin 2\pi fT}{1 + \cos 2\pi fT} = \frac{2\pi fT}{2} \tag{6}$$

and the phase of the multipath operator increases linearly. When a is not equal to one oscillations in the phase angle appear, (in addition to the linear term of (6)) and, as with the amplitude spectrum, they have period $\Delta f = 1/T$.

Thus in both the amplitude and phase spectrum the effect of the multipath operator is to introduce a periodic modulation whose period in the frequency domain is equal to the inverse of the delay between the interfering events, and whose amplitude depends on their relative amplitude.

If the modulation could be removed from the amplitude and phase spectra, and those spectra inverse transformed, the result would be the original waveform S(t) without multipathing. Conversely, from the period and amplitude of the modulation we can obtain the time shift and scale factor for the second event. The ordinary way to remove unwanted periodic components from the spectra is by filtering, and that is what is done in this case.

2. Filtering the Spectrum

There are some useful analogies between cepstral filtering to remove the effect of the multipath operator, and filtering a time series in the frequency domain, if the roles of time and frequency are interchanged. The Fourier transform of the complex logarithm of the spectrum is called the complex cepstrum, in which time rather than frequency is the independent variable. Components of the spectrum periodic in frequency give rise to peaks in the cepstrum just as periodic components of a time series give rise to peaks in the spectrum. Removing these peaks from the cepstrum removes their corresponding periodic component from the spectrum.

The complex cepstrum is the sum of the Fourier transform of the logarithm of the amplitude spectrum and the Fourier transform of the phase spectrum. Due to the symmetry of the amplitude and phase spectra the complex cepstrum is a purely real function. It is found most convenient to separately consider these transforms, called the real and imaginary cepstra, than to add them.

Two general ways to filter the cepstrum, analogous to high- and-low pass filtering and to bandpass filtering, are used in this study. The low pass filter zeros all points in the cepstrum beyond some time less than the cepstral peak it is trying to eliminate. High pass filtering retains these points while zeroing points at short times. Bandpass filtering zeros a few points centered on the cepstral peaks, or all points except those near the peaks. For short period data it is found that the bandpass filter is more effective than the high or low pass filter.

A simple calculation will show that if the interfering signals are identical the cepstral peak consists of an isolated point. Since real interfering signals are not precisely identical their peaks will not be so sharp and it is desirable to zero one or perhaps two points on each side of the peak to achieve the best separation.

Peaks in the cepstrum should be restored to the value they would have had if there were no multipathing. Since this is not known, they are set to zero, creating a disturbance in the cepstrum. The effect of the multipathing cannot be perfectly removed from the signal for this reason.

After zeroing the peaks of the cepstra, the direct transform back to the frequency domain is taken. Within the limitation mentioned above, the spectrum is now free of modulation, and represents only the unmultipathed signal. After taking the antilogarithm of the amplitude spectrum, an inverse transform to the time domain, called the shortpass output, recovers the original signal. When this is subtracted from the input trace, the remainder is the second arrival due to the multipath operator.

If we retain only the cepstral peaks, and transform to the frequency domain, we will be left with modulation of the log amplitude spectrum. To obtain the long pass output, we first take the antilogarithm, and then the direct Fourier transform to the time domain.

The mean level of the modulation is zero, because the value of the cepstrum at the first point was set equal to zero. When the antilogarithm is taken, the mean value becomes one. The direct transform of this constant term is a point of amplitude one. There is also point in the long pass output due to the modulation. The modulation has period 1/T, so its transform is a point delayed by T past the point due to the original signal. Since there is a linear term in the phase spectrum, due to the reverse rotation necessary to correctly position the short pass output, both points are shifted to larger times by a amount roughly equal to half the length of the original signal.

### 3. Interpreting the Results

In practice the real and imaginary cepstra are not smooth functions as implied by the previous paragraphs. Noise, present to some degree in all records, is by its nature non-repetitive, so it does not produce modulation in the spectra. Rather, it leads to noise in the spectra and noise in the cepstra. Its net effect is of course to obscure the peaks due to real arrivals.

Once the real and imaginary cepstra have been calculated and displayed, the analyst must choose the peaks he wishes to filter out, which on some records may be no larger than those due to noise. A useful test is to compare the real and imaginary cepstra. The occurrence of peaks at the same time in both cepstra is an indication that an arrival occurs there. However, it is usually a matter of experience and trial and error to successfully separate multipathed signals. The correctness of the filter points must be judged by the quality of the results. Some guides for such judgement are given here.

It is observed that when filtering is done correctly the short pass output is very small until the time corresponding to the filter point. If there is no real arrival at this time, the first motion will not be at the filter time but earlier.

If a clean first motion is present one next looks for similarity of motion between the output traces. Peaks separated by the same time delay as the filter time should correspond to one another. If these tests are met, a negative test must also be passed. The outputs must not show oscillations 180° out of phase for any great part of the motion. Such oscillations mean that the technique is creating motion where there was none actually recorded.

## C. EXPERIMENTAL RESULTS

To test the cepstrum's ability to separate identical interfering signals, representative waveforms recorded on short period instruments at NORSAR were shifted in time with respect to themselves, multiplied by a scale factor, and added to themselves. The multipath operator was known exactly in this case, and the performance of the technique could be readily evaluated.

Figures II-1 through II-4 show results obtained using event KAZ/170/04N, a presumed underground nuclear explosion from eastern Kazakh. Figure II-1 is the waveform, as recorded with good signal-to-noise ratio at a rate of 10 samples per second. Figure II-2 shows the result of shifting the waveform of Figure II-1 by .7 seconds, multiplying it by .5, and adding to the original signal. The result is a complicated trace with no obvious second arrival.

Both the real and imaginary cepstra showed a clear peak at a delay time of .7 seconds for this synthetic event. A bandpass filter which

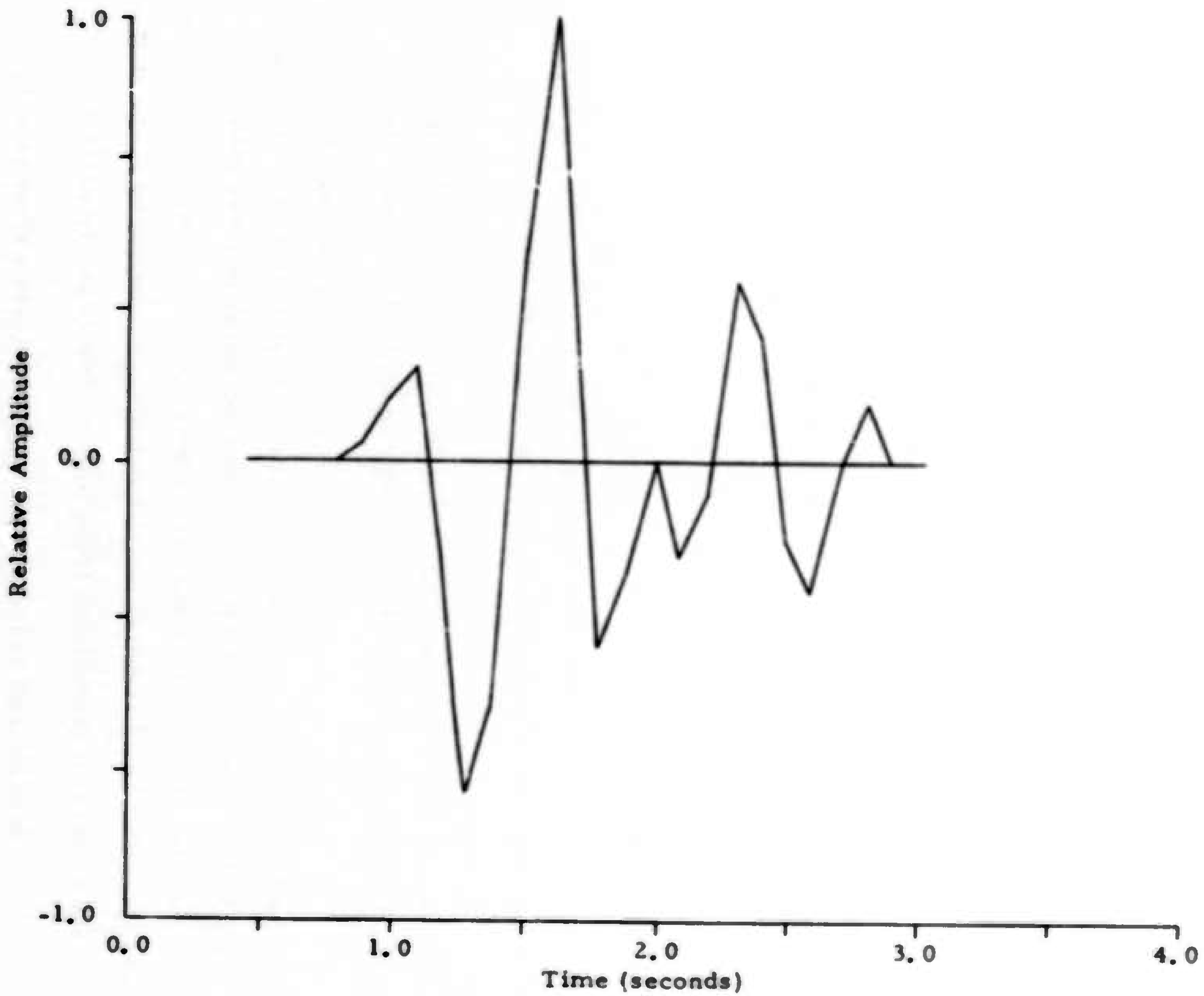


FIGURE II-1

WAVEFORM FROM KAZ/170/04N

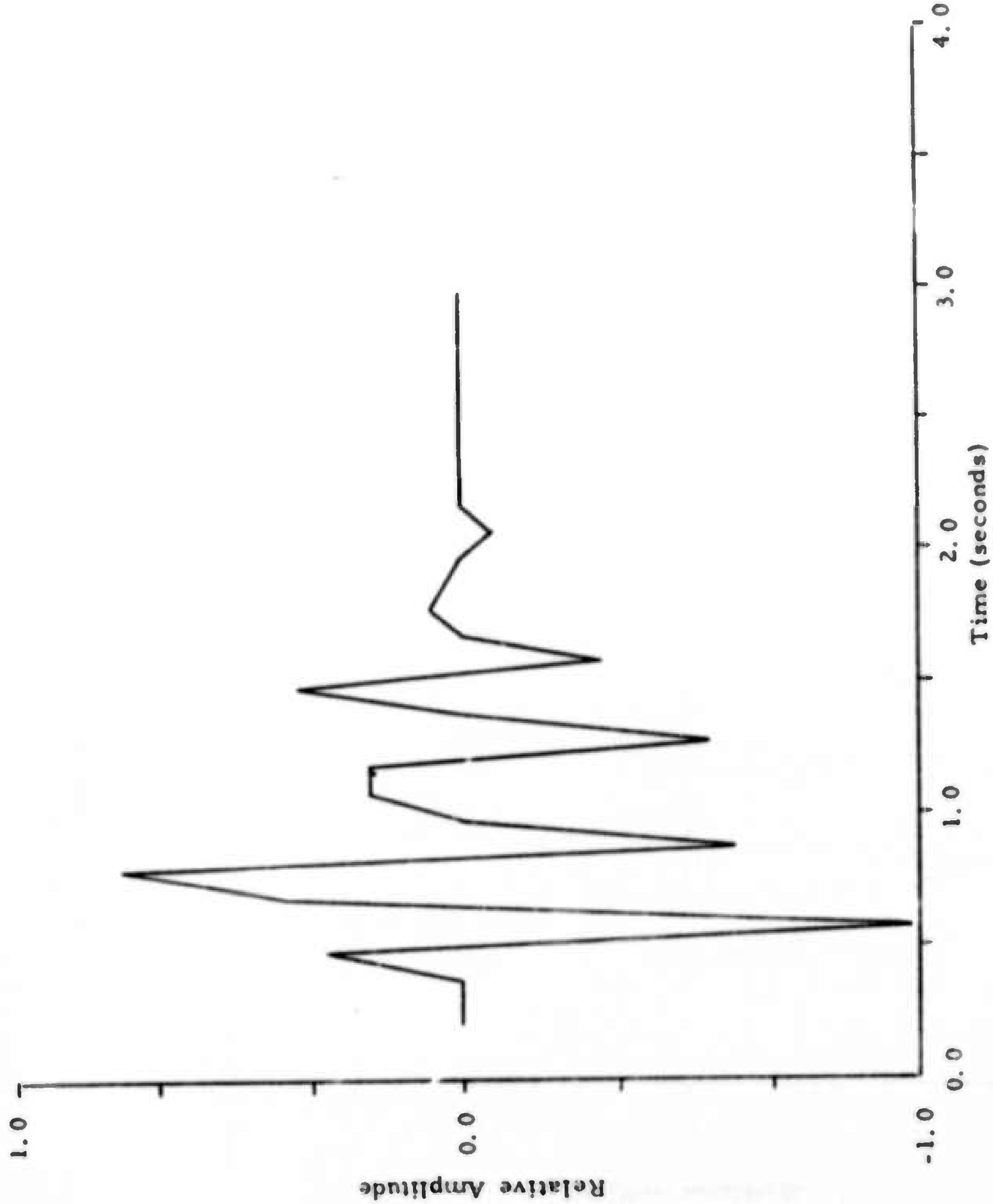


FIGURE II-2

WAVEFORM FROM KAZ/170/04N
SCALED BY .5 AND SHIFTED BY .7 SECONDS

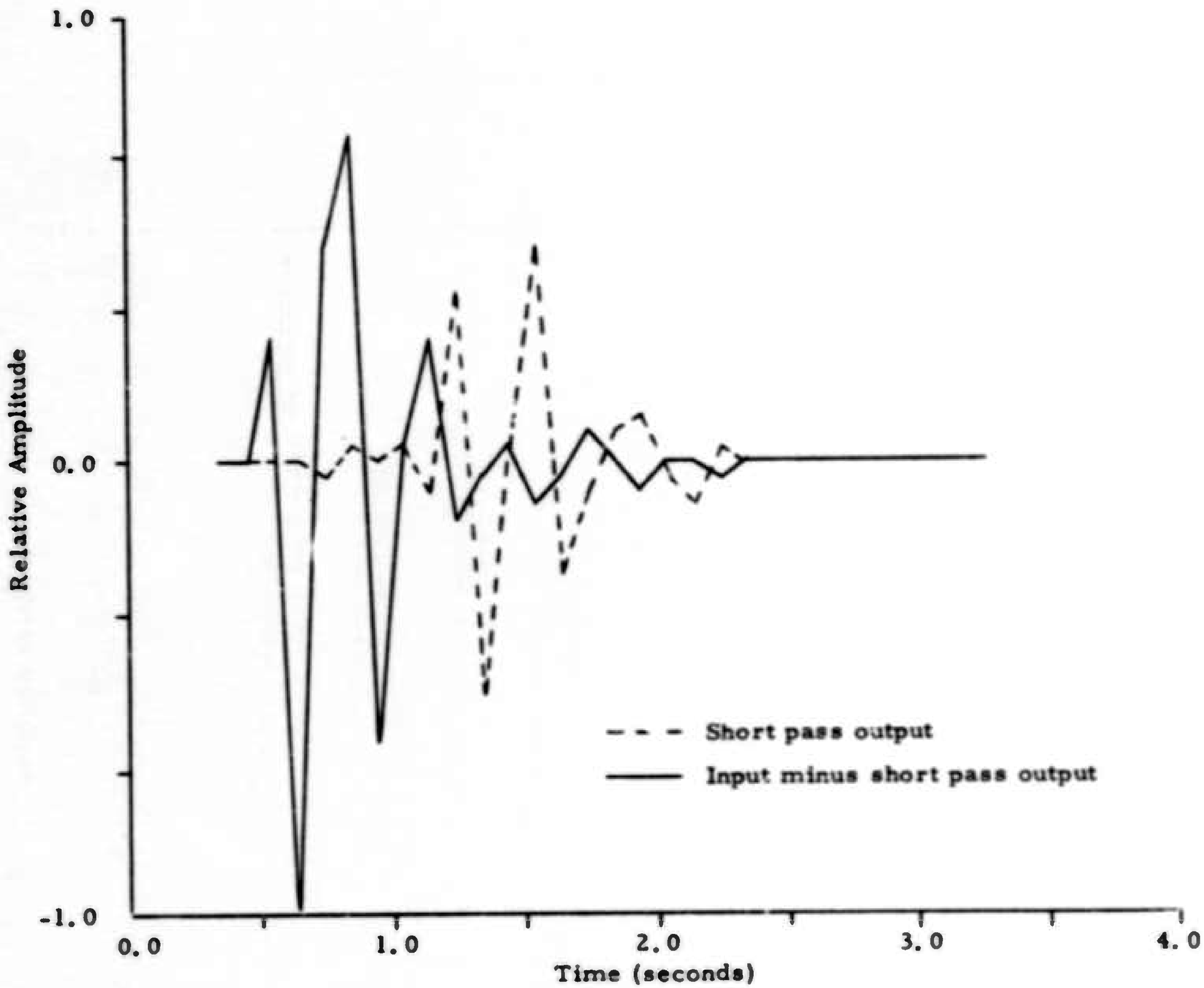


FIGURE II-3

SHORT PASS OUTPUTS FROM KAZ/170/04N
SCALED BY .5 AND SHIFTED BY .7 SECONDS

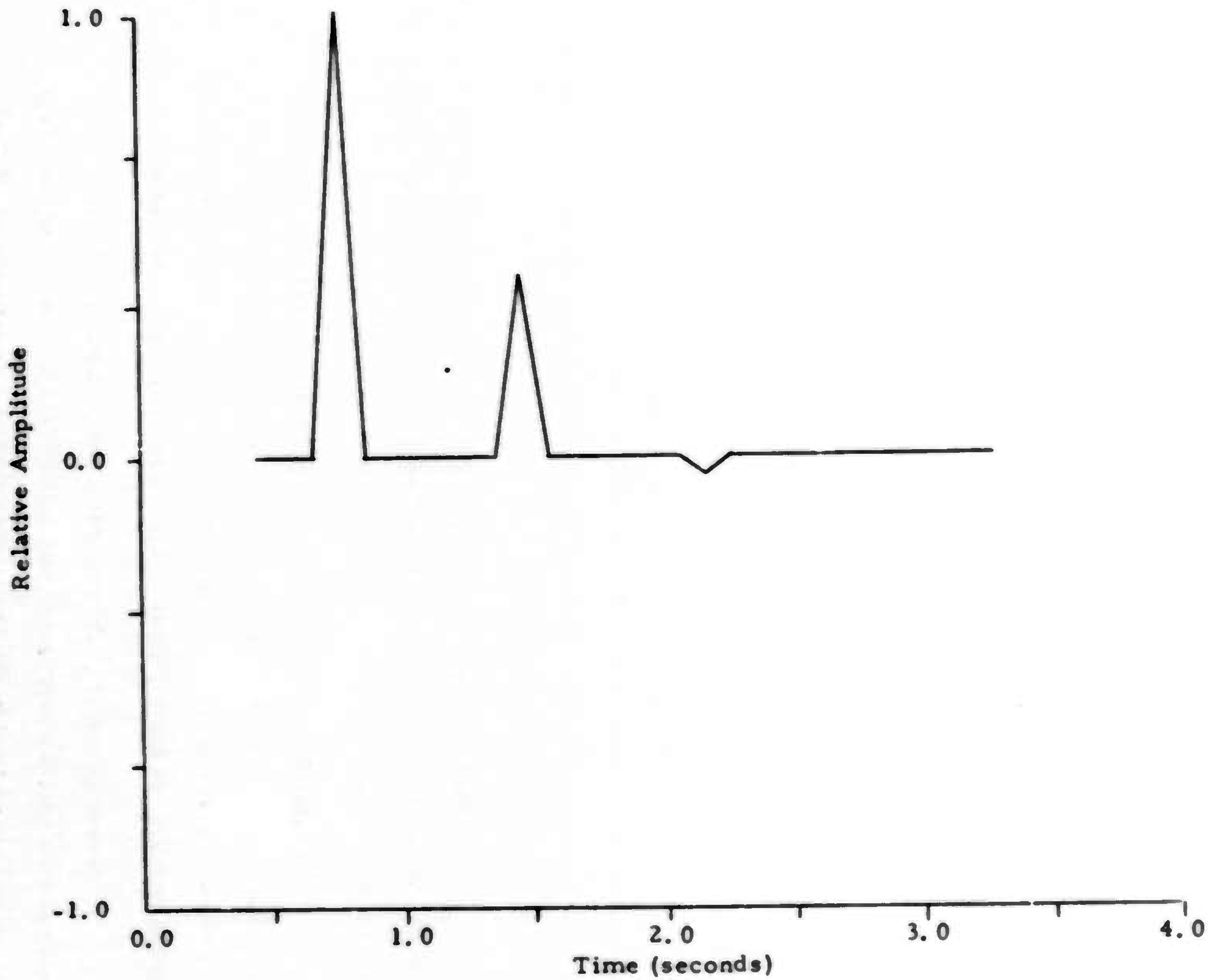


FIGURE II-4

LONG PASS OUTPUT FOR KAZ/170/04N
SCALED BY .5 AND SHIFTED BY .7 SECONDS

removed the points at .7, 1.4 ... seconds was applied to both cepstra, as discussed in Subsection B.

The short pass output, and the input minus the short pass output, are shown superimposed in Figure II-3. The resemblance of the waveforms to one another, and to the original trace of Figure II-1, is clear. Furthermore, the second trace is indeed delayed by .7 seconds with respect to the first, and is roughly half the amplitude of the first.

The long pass output, the estimate of the multipath operator, is shown in Figure II-4. The amplitude of the second peak, delayed by .7 seconds with respect to the first, is very close to half the amplitude of the first. It is often observed that the relative amplitudes of the long pass peaks are closer to the correct values than those in the short pass output.

Next, in Figure II-5, is shown the waveform of Figure II-1 delayed by .7 seconds, multiplied by 2.0, and added to itself. Again the composite waveform is complicated. The filtered waveforms are shown in Figure II-6, and the long pass output in Figure II-7.

The results in this case are not as clear as in the first example, but the similarity between the filtered outputs is still noticeable. The long pass output is again more clear than the short pass output.

Schafer (1969) suggests that the complex cepstrum technique is generally more effective when the multipath operator is minimum phase. When the second signal has larger amplitude than the first, as in this example, the multipath operator is not minimum phase, and we expect less resolution.

Investigation of KAZ/170/04N, and of KAZ/157/03N, a presumed underground nuclear explosion, and VAN/072/23N, an earthquake near Vancouver Island, have lead to the conclusion that at a separation of .7 seconds,

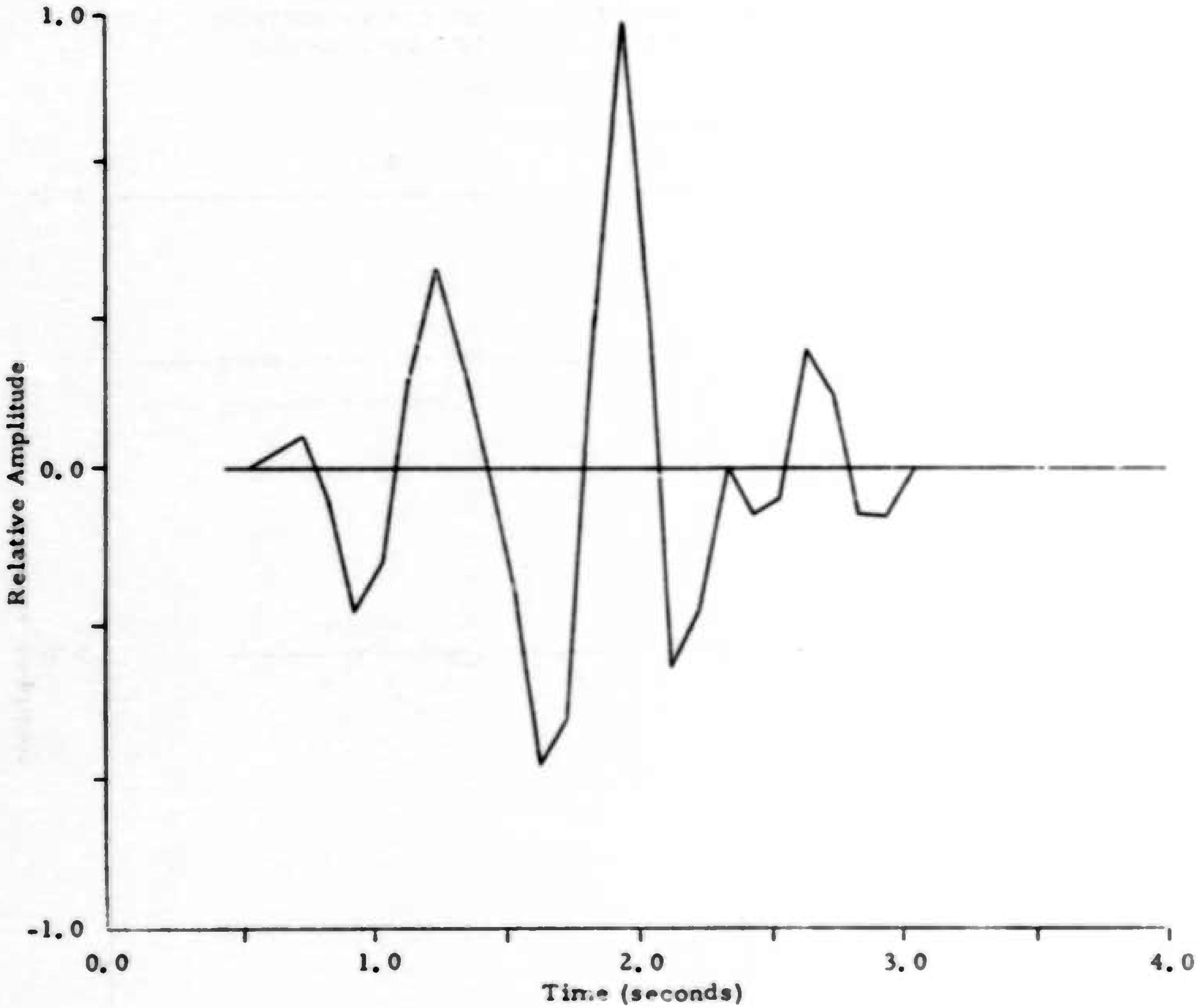


FIGURE II-5

WAVEFORM FROM KAZ/170/04N
SCALED BY 2.0 AND SHIFTED BY .7 SECONDS

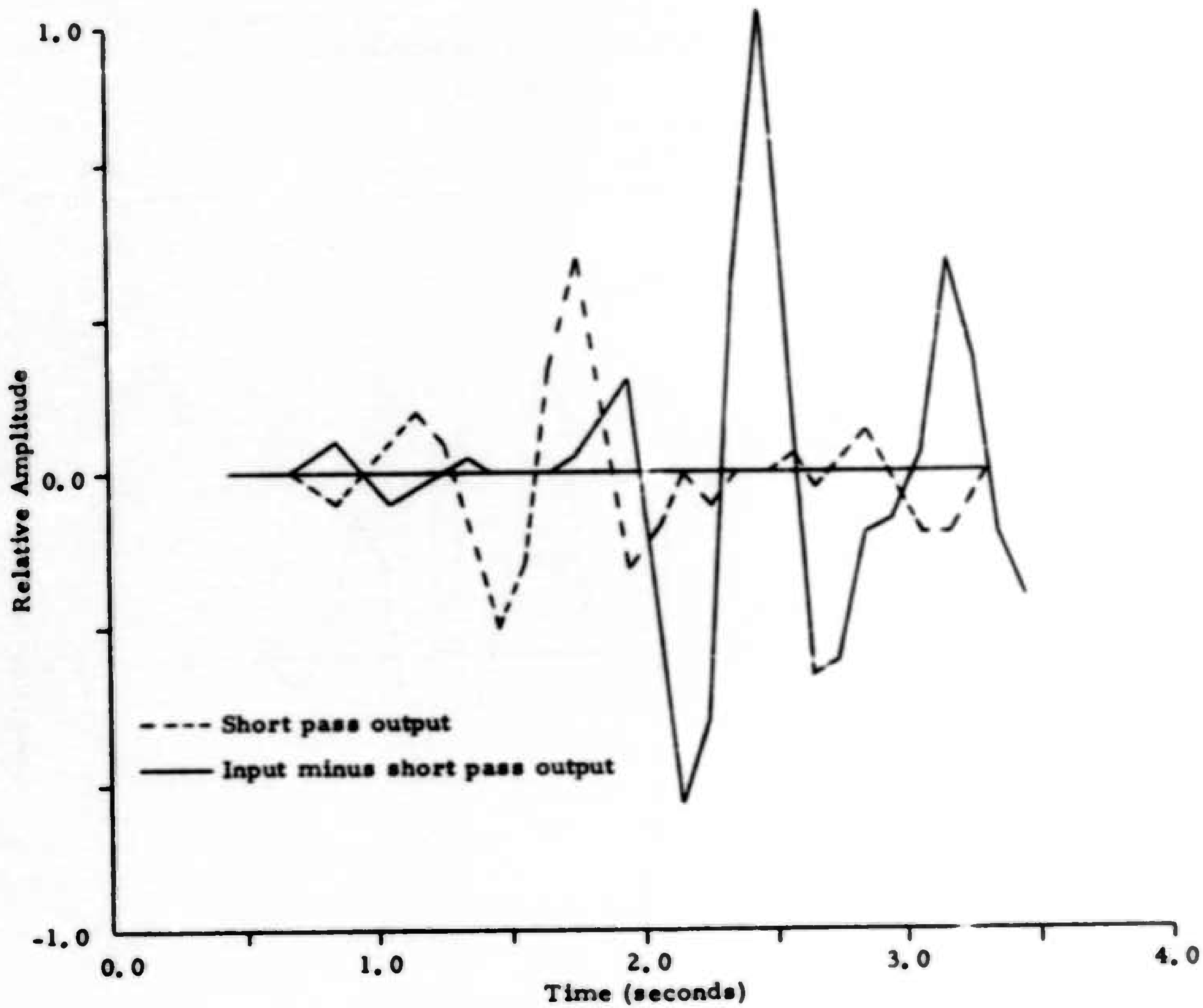


FIGURE II-6

SHORT PASS OUTPUTS FOR KAZ/170/04N
SCALED BY 2.0 AND SHIFTED BY .7 SECONDS

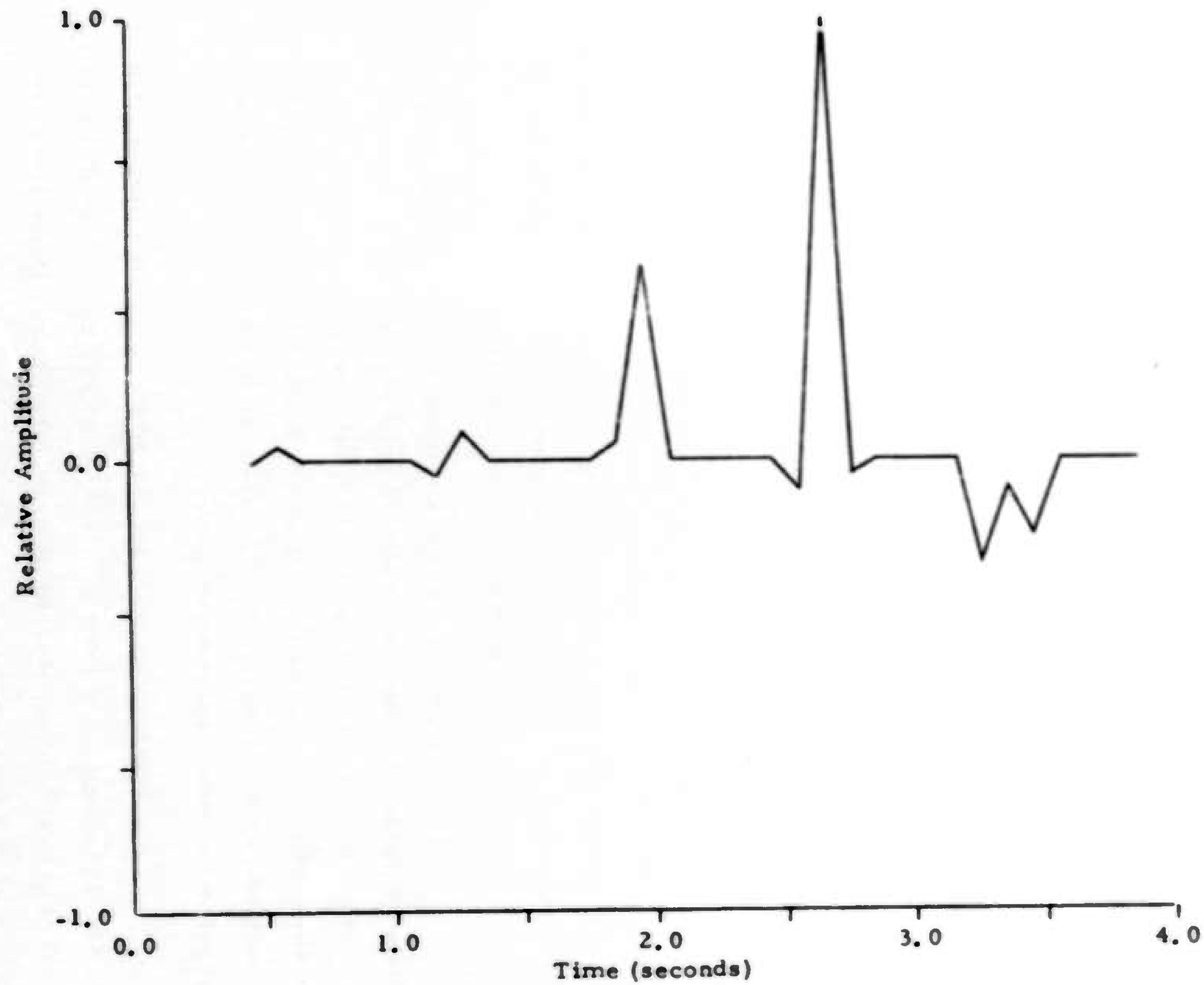


FIGURE II-7

LONG PASS OUTPUT FOR KAZ/170/04N
SCALED BY 2.0 AND SHIFTED BY .7 SECONDS

second events of relative amplitude from about 2.0 down to a level where the second signal disappears in the noise are separable. The quality of separation is comparable to that of Figures II-3 and II-6, with larger second arrivals generally resulting in poorer resolution of the signals.

In the absence of an objective criterion as to when two signals are resolved, it cannot be said precisely when the technique fails. However, identification of the second signal becomes very dubious when its amplitude is more than twice that of the first, at this separation in time.

For these waveforms, even a tenth of a second reduction in delay time makes the signals unresolvable. We may conclude that .7 seconds is the limit of resolution for these events.

For delays of the order of 1.5 seconds or more the signals are often separable by eye on the unprocessed trace, for reasonable scale factors. For delays between .7 and 1.5 seconds the upper limit on the relative amplitude for successfull resolution increases somewhat, but probably does not exceed four. The lower limit depends on the noise as before.

With these results in hand we turn to a real event, KAZ/115/03N. This was a presumed explosion, detonated on April 25, 1971, at 3 hours 32 minutes, 58 seconds, in Eastern Kazakh, with bodywave magnitude 5.9 and surface wave magnitude 4.3, reported by NORSAR (Filson and Bungum, 1971). These magnitudes place it outside the main group of central Asian explosions, although not in the central Asian earthquakes.

The waveform as recorded at NORSAR is shown in Figure II-8. Both the real and imaginary cepstra are shown in Figure II-9. A small but fairly clear peak is seen at 1.1 seconds time delay on both traces. Other peaks are present on each, but not on both.

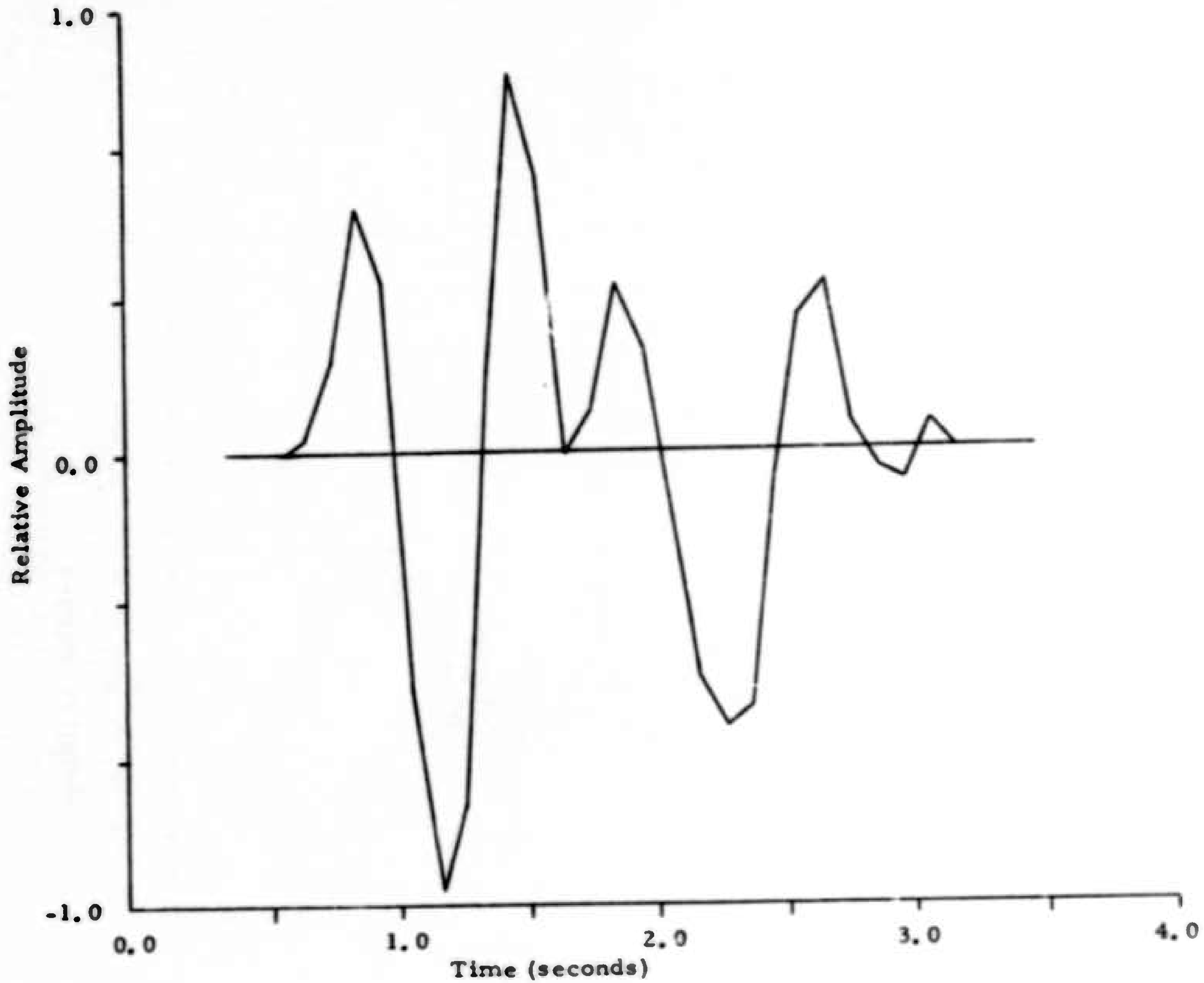


FIGURE II-8

WAVEFORM FROM KAZ/115/03N

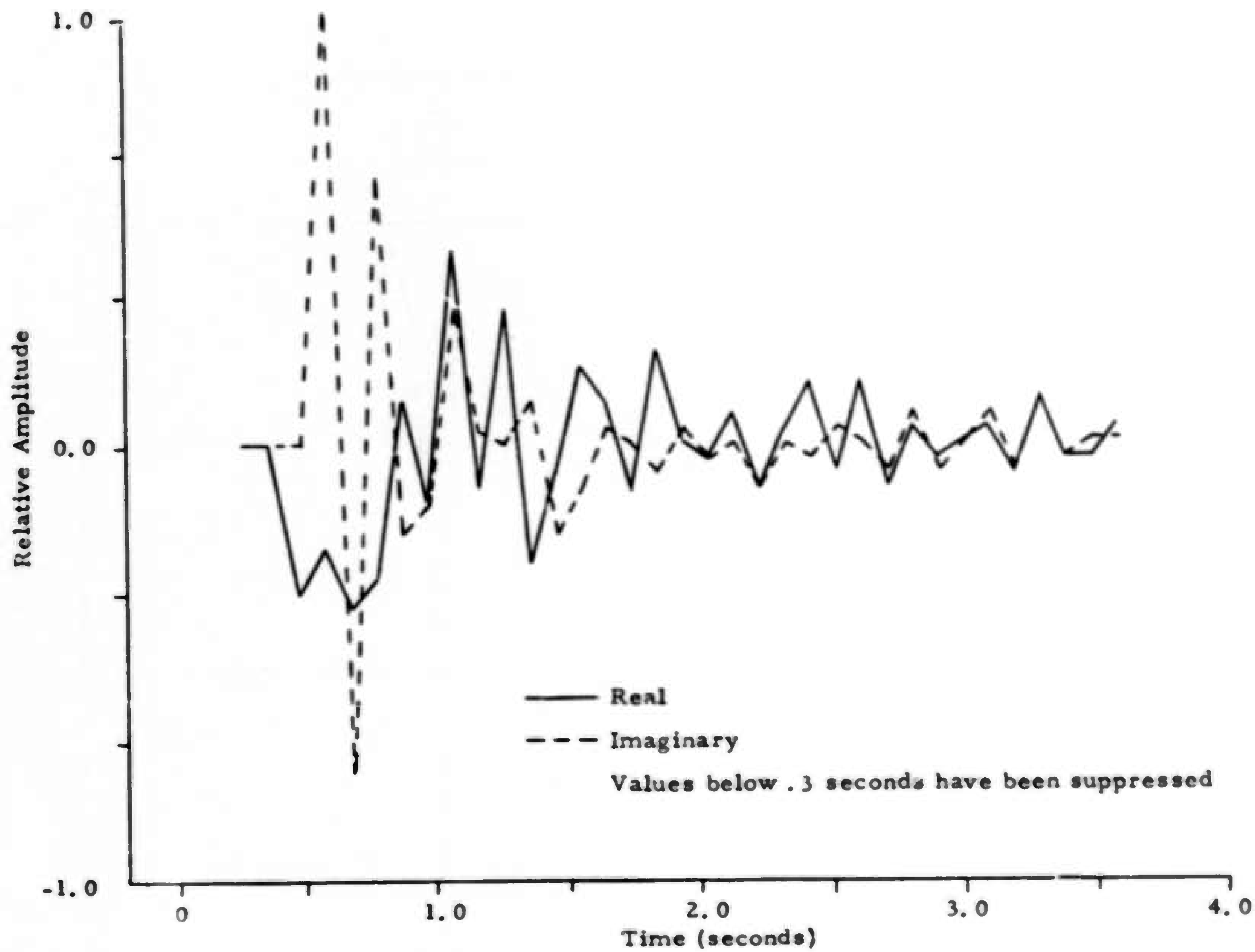


FIGURE II-9

REAL AND IMAGINARY CEPSTRA FROM KAZ/115/03N

A bandpass filter removing only the point at 1.1 seconds results in the output waveforms shown in Figure II-10. The lack of large motion in the second signal before 1.1 seconds after the start of the first signal is noticeable. Furthermore, the first four extrema in the first waveform occur just 1.1 seconds before the first four extrema in the second waveform, and corresponding peaks are in roughly constant proportion to one another. At the end of the waveform these similarities are lost as noise obscures the motion.

In the long pass output, shown in Figure II-11, two clear peaks are visible. The first has amplitude one and the second, following it by 1.1 seconds, has amplitude .30. Another peak 1.1 seconds later has small amplitude. Thus the evidence suggests the existence of sequential underground nuclear explosions whose waveforms are separated in arrival time by about 1.1 seconds, and whose body wave amplitudes are about in the ratio 3:1. Because the motion has the same sign for corresponding peaks the second event is not due to a surface reflection.

The relative magnitude of the presumed second event as reported here is not sufficient to account for the departure of .3 magnitude units from the explosion population reported by Filson and Bungum(1971). However, the presumed events need not have been detonated at the same location, and differences in effeciency in generating body waves might account for the difference in magnitude.

Whether these waveforms are due to two similar events is not a question the cepstrum technique is competent to answer. The claim made here is that the waveforms exhibited in Figure II-10 sum to the observed waveform, and that the two individual waveforms are similar. Positive identification of the waveforms with separate presumed explosions is a step for the analyst to take.

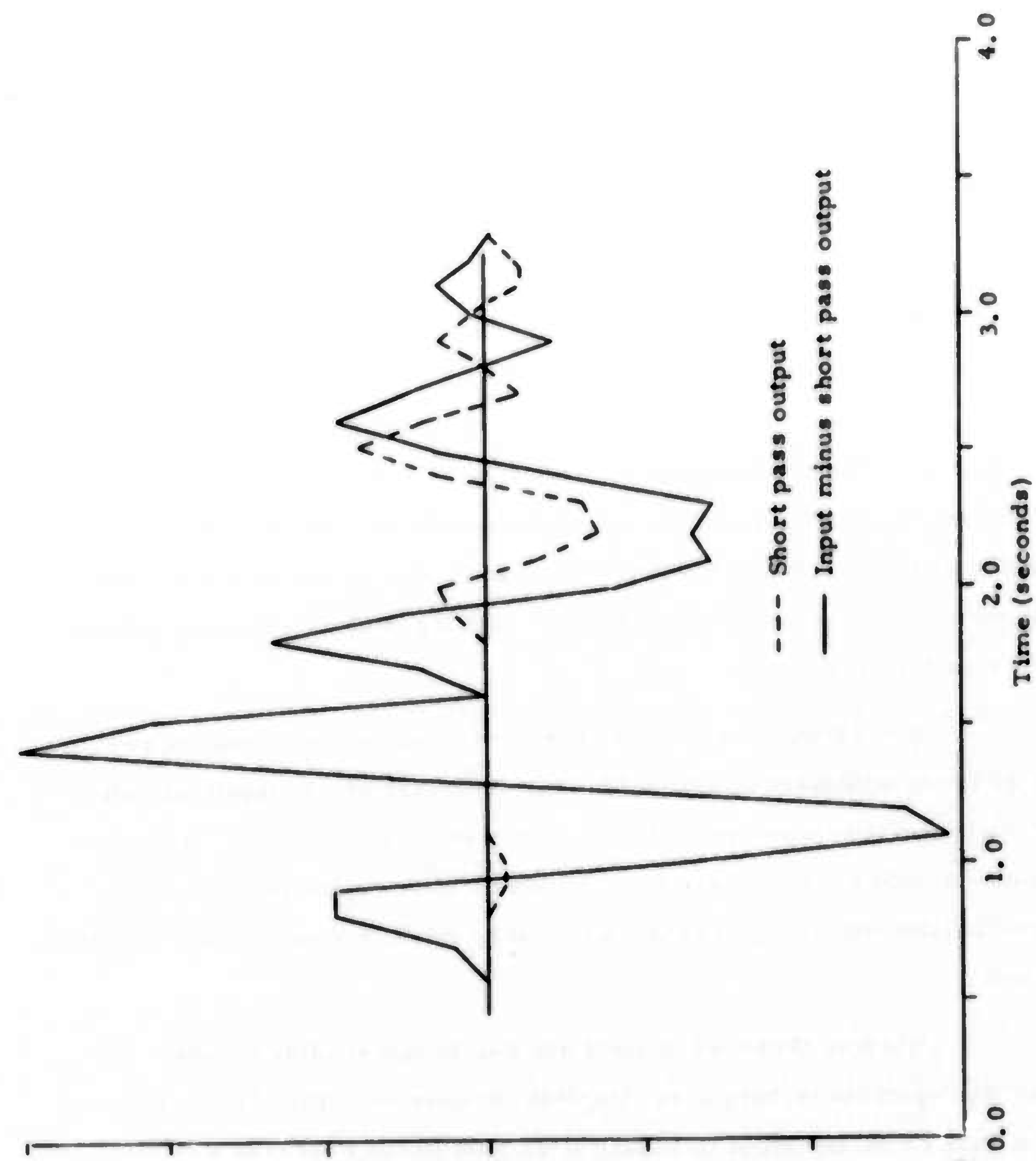


FIGURE II-10

SHORT PASS OUTPUTS FOR KAZ/115/03N

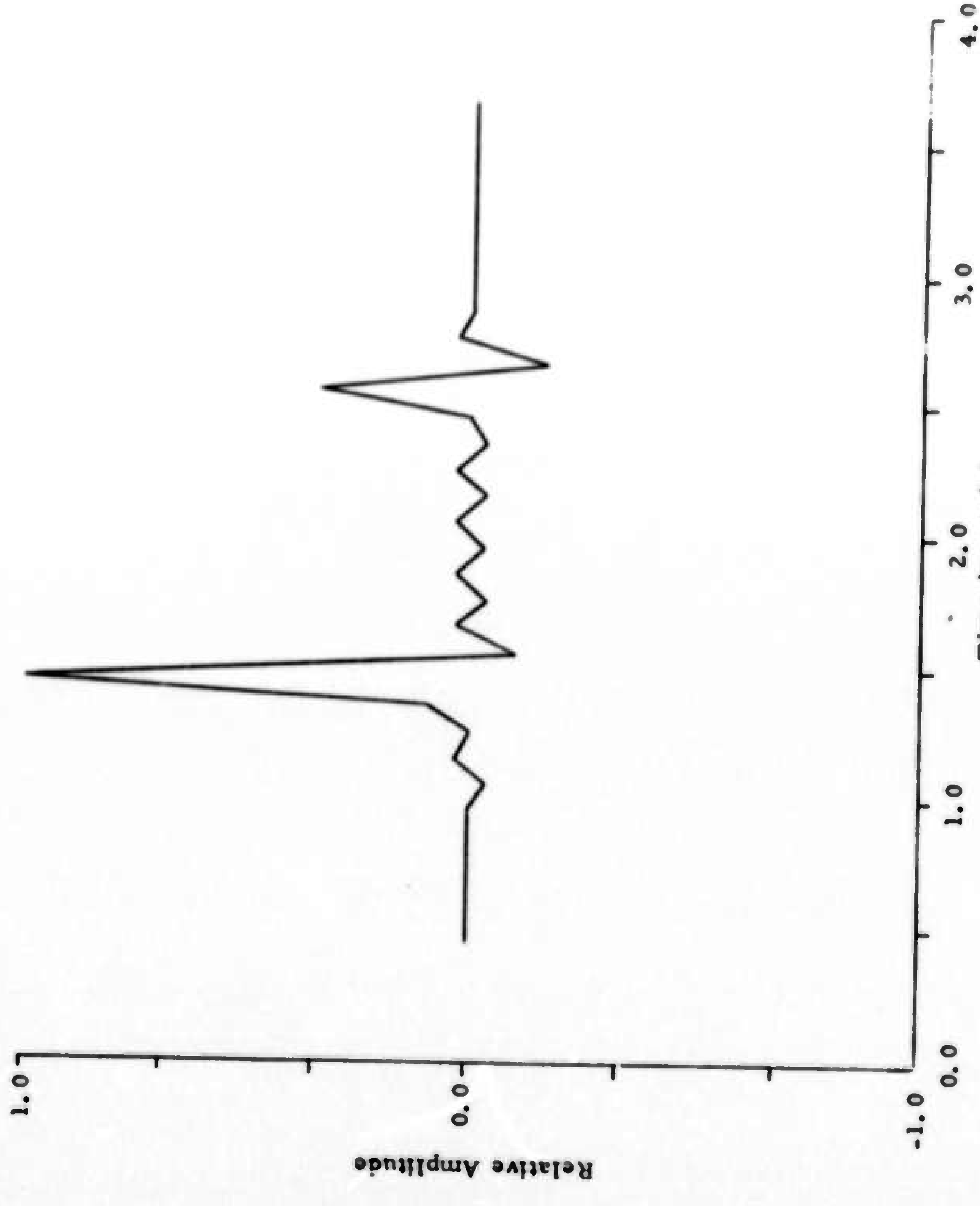


FIGURE II-11

LONG PASS OUTPUT FOR KAZ/115/03N

## SECTION III
## EIGENSPECTRUM TECHNIQUE

### A. INTRODUCTION

About 15% of all long period seismic events are "interfering", in that they consist of a mixture of two different events from different azimuths. Beamsteering can separate these events to some extent, but if one is much smaller than the other, it will be hidden within the sidelobes of the main event. The eigenspectrum technique offers a possibility of separating such events with greater resolution.

In the eigenspectrum technique the data matrix is formed and its eigenvalues and eigenvectors computed. It is shown that the eigenvalues are proportional to the signal powers and that the eigenvectors lie in the direction of the signal vectors. The orthogonality of the eigenvectors makes possible the calculation of the signal powers with great accuracy.

### B. DESCRIPTION OF THE TECHNIQUE

#### 1. Theoretical Basis

For an array of sensors, the signal vector is defined to be

$$S = \begin{bmatrix} \vdots \\ F_i(f) \\ \vdots \end{bmatrix} \qquad (7)$$

where $F_i(f)$ is the Fourier transform at frequency f of the motion at the i th site. From S the data matrix can be formed as

$$\Omega = SS^H = \begin{bmatrix} \vdots & \cdot & \vdots \\ & F_i(f)\,F_j(f) & \\ \cdot & \cdot & \cdot \end{bmatrix} \qquad (8)$$

where H denotes Hermetian transpose.

There are N eigenvalues of this matrix, where N is the number of channels, and they are $S^H S, 0\ , 0\ .\ .\ .$ Inspection of 7 shows that $S^H S$ is N times the average power at a site. The eigenvector associated with this eigenvalue is V.

If only one signal, of amplitude A and wavenumber $\vec{k}$ is present at sites $\bar{X}_i$, then

$$S = V = A \begin{bmatrix} \vdots \\ e^{i\vec{k}\cdot\vec{X}_i} \\ \vdots \end{bmatrix} \qquad (9)$$

and $V^H V$ is the power of this signal. Also, the eigenvector V is in the direction of the wavevector $\vec{k}$. The more interesting case is where two signals V and W are present, and then

$$\Omega = VV^H + WW^H + VW^H = WV^H \qquad (10)$$

The second two terms are due to correlation between the signals. Because of their presence, the eigenvalue is

$$V^H V + W^H W + W^H V + V^H W \qquad (11)$$

and is thus N times the total power. The eigenvector is V+W, and does not indicate the direction of propagation.

The cross correlation terms may be eliminated by averaging the data matrix $\Omega$ over frequency. If the signals are uncorrelated, the average of $VW^H$ and $WV^H$ will be zero;(this is the definition of uncorrelated signals). Under these conditions the average data matrix becomes

$$\bar{\Omega} = \overline{VV^H} + \overline{WW^H} \qquad (12)$$

The eigenvalues of this matrix are not so easy to interpret as when the data matrix represented a single frequency. We might hope that the eigenvalue of the average matrix is the average of the eigenvalues of the individual matricies and thus proportional to the average power. However, this is so only when the eigenvectors are all parallel. Parallel does not mean pointing in the same direction in wavenumber space but:

$$V(f_1)\ V^H(f_2) = |V(f_1)|\,|V(f_2)| \tag{13}$$

If we use (9) for the signal vector, the left hand side of (13) is:

$$A(f_1)\ A^*(f_2) \sum_{i=1}^{N} e^{i(\vec{k}(f_1)-\vec{k}(f_2))\ \vec{X}_i} \tag{14}$$

where * denotes complex conjugate
and the right hand side is:

$$N\,|A(f_1)|\,|A(f_2)| \tag{15}$$

and these quantities are not equal, although they approach equality as $f_1$ approaches $f_2$.

Thus to obtain a data matrix which is free of cross terms we must average over a band of frequencies, but in doing so the interpretation of the eigenvalue as the average power is lost. In a practical situation we average over a band wide enough to eliminate most of the cross correlation but narrow enough to not change the eigenvalues too much from the average power.

The case where random noise is present is easily incorporated into the theory. Random noise is represented as usual by a signal vector whose components have random amplitudes and phases. If such a component is $n_k e^{i\phi_k}$, then the j-k th component of the data matrix is $n_j\ n_k^* e^{i(\phi_j-\phi_k)}$. Averaging over frequency reduces those components for which $j \neq k$ to zero. On diagonal components are $n_j n_j^*$. The average over the frequency band gives the same value at each site. Thus the noise contribution is $\beta I$ where I is the identity matrix. Then the data matrix for one signal in noise with which it is uncorrelated is:

$$VV^H + \beta I \tag{16}$$

and it can be seen that the eigenvalue of this matrix is $V^H V + \beta$, while the eigenvector is V. Thus the estimate of the direction of the incoming signal is not in concept influenced by noise.

2. Details of the Calculation

The advantage of the eigenvector technique is that the eigenvalues can be calculated independently of one another, within limits of the calculating machinery. Thus one signal is not obscured in the sidelobe of another's beam pattern.

The particular eigenvalue extraction technique used in this study is called deflation. The first and largest eigenvector of a matrix is found by starting with a unit trial vector X. The quantity:

$$\Omega X = aY \tag{17}$$

is calculated, where Y is also a unit vector. Each component of $(X - Y)$ is compared to some small test value. If the difference is less than the test value for all components, $X \cong Y$ is an eigenvector and a is its eigenvalue. If it is not, Y is taken as the new trial unit vector, and the process is repeated. Under certain circumstances the technique will fail to find an eigenvalue, but ordinarily convergence is achieved in less than 20 iterations.

The data matrix is now deflated by calculating

$$\Omega' = \Omega - aYY^H \tag{18}$$

If the calculated eigenvalue and eigenvector are close to their true values, the contribution to the matrix from the largest signal will be almost completely removed, and calculation can proceed in the same way to find the next largest eigenvalue and associated eigenvector.

### 3. Errors

Errors can enter the calculation at a number of points. As mentioned in subsection A, the necessity for averaging the data matrix over frequency is incompatible with the identification of the eigenvalue with the average power. This error can be minimized by correct choice of the frequency averaging band.

Another kind of error arises in the deflation process. The calculated eigenvector and eigenvalue will not be exactly equal to their true values due to limitations imposed by the calculating equipment. Therefore the deflation step, shown in (18), will leave some error matrix in addition to the matrix containing the next largest eigenvalue. If the next eigenvalue is relatively small, this error matrix can significantly distort the signal power estimate.

Finally, as a signal power approaches the noise level, the estimate of the signal also approaches the noise power. The degree to which this is a problem depends on the precision with which we wish to estimate the signal power, as can be seen from (16). In this case, the error in the eigen-spectrum technique is the same as in beamsteering.

### 4. Calculations on Synthetic Data

The work reported here is based on synthetic data. Chirp waveforms were used to construct a signal at each site of a hexagonal array. The waveforms all had the form

$$X(t) = \frac{A}{2} \sin\left(2\pi f_o t + \frac{2\pi \Delta f t^2}{T}\right)\left(1 - \cos\frac{2\pi t}{T}\right) \tag{19}$$

where A was the amplitude, $f_o$ the starting frequency, and $\Delta f$ the frequency change due to dispersion. The length of the waveform was T, and the final multiplicative term ensured that the amplitude rose from zero to A and then fell back to zero.

Each waveform was Fourier transformed, and the sum of the squares of the Fourier coefficients was averaged over some frequency interval, giving the average power for that interval. Then at each site the composite waveform, the sum of as many chirp waveforms as desired, each with the proper time delay, was generated. A data matrix for each frequency was set up by calculating the Fourier transform and forming the products defined by (9). These data matrices were averaged to yield the average data matrix defined by (12). A random noise matrix of the form $\beta I$ was added.

The largest eigenvalue was extracted from the data matrix, and divided by the number of sites. This was the technique's estimate of the power, to be compared with the power found by Fourier transforming the original signal. The eigenvector associated with this eigenvalue was multiplied by a look direction vector, and the look direction rotated. When this product attained its largest value the look direction coincided with the signal direction.

## C. EXPERIMENTAL RESULTS

The signals described in Subsection B were generated at each site of a seven element hexagonal array of 50 kilometer apeture, similar to the inner ring of sensors at the Alaska Long Period Array. The signal lengths were different in order to ensure that the signals were uncorrelated. A model consisting of a 700 second signal and a 150 second signal, plus random noise, was usually assumed. Signals started at .025 Hz and ended at .060 Hz. The azimuth of one signal relative to the other was varied, as were their relative powers, and the noise power.

Obviously the technique can never recover the original signal power and azimuth exactly. It was decided that if the estimated power was within .5 dB of the true power, and the estimated azimuth within $10^{\circ}$ of the true azimuth, then the signals were separated. If either of these conditions were not met, the signals were not resolved. The tolerance on the power estimate is closer than the azimuth tolerance because this technique is expected to be used to estimate powers of signals whose azimuths are already known from measurements with short-period instruments.

When the noise power was set equal to zero, it was found that a signal about 27 dB below a larger signal could be resolved over a wide angular range. As the noise power was increased, no change in detectability was found until the noise reached -30 dB relative to the larger signal. Then the estimate of the smaller signal began to be affected by the noise. As the noise power was raised, the level at which the smaller signal could be detected also rose, staying 3 dB above the noise.

It was decided to keep the noise power at -30 dB with respect to the larger signal. Lowering the noise power would not increase the region of resolution of the signals, and raising it decreased that region.

Figure III-1 shows half of a plot of the region of detectability, a small signal buried in a large signal, with the noise 30 dB below the larger signal. The other half of the plot, from $180^{\circ}$ to $360^{\circ}$, was symmetrical with Figure III-1. The horizontal axis is the true azimuthal separation between the two signals, the vertical axis is their relative amplitude. Signals falling within the shaded region were resolved according to the criteria stated above.

As the smaller signal decreased in power towards the noise level, the eigenspectrum's estimate of power approached the noise power asymptotically. As the azimuthal separation between the signals decreased, more complicated behavior was observed. The estimate of the azimuth closely agreed with the true azimuth as the separation decreased, until the separation reached about $90^{\circ}$. Further decreases in the true separation resulted in errors in the

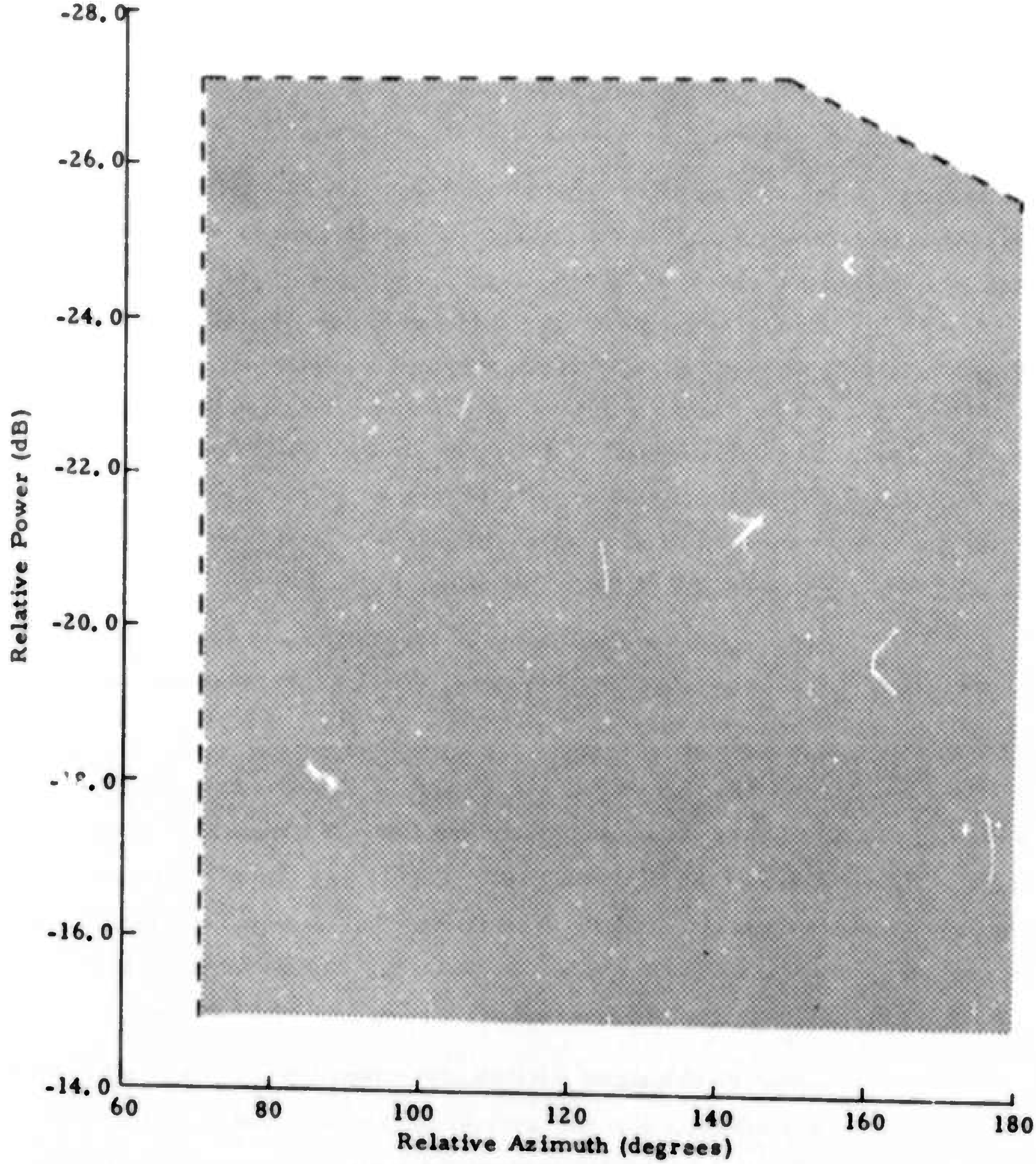


FIGURE III-1

REGION OF SEPARATION FOR INTERFERING CHIRP WAVEFORMS estimated seperation of up to $10^{\circ}$. The estimated separation never decreased below $70^{\circ}$, and below $70^{\circ}$ true difference in azimuth, the estimate of signal power also moved out of the tolerance limits for detection. The cause of the slight dip in detection level around $180^{\circ}$ is unknown.

Behavior at relative amplitudes of less than -14 dB was not investigated. However, beamsteering would separate such signals, and there is no reason to believe that the eigenspectrum technique would not also.

Figure III-2 shows the eigenspectrum for the largest signal in a model whose parameters are:

| Component | Azimuth | Power |
|---|---|---|
| Signal 1 | $135^{\circ}$ | 0 dB |
| Signal 2 | $295^{\circ}$ | -26 dB |
| Noise | Isotropic | -30 dB |

The eigenspectrum is the product of the eigenvector with a look direction vector. If the signals are separated this product will have one peak at the signal azimuth and no other peaks. The eigenspectrum in Figure III-2 peaks at $135^{\circ}$, just the signal azimuth. The eigenvalue is almost identical to the signal power.

Figure III-3 shows the eigenspectrum for the second eigenvalue. It peaks at $300^{\circ}$, an error of $5^{\circ}$ in azimuth, but well within the criterion for resolution. The eigenvalue lies 25.5 dB below the first eigenvalue, also within the criterion for resolution.

A third eigenvalue was extracted from the data matrix, although only two signals were modeled. Its eigenvalue lay 36.4 dB below the first eigenvalue, and its eigenspectrum is shown in Figure III-4. The presence of two peaks indicates that the eigenvector is not of the form of a signal vector. Consequently, this eigenvector and eigenvalue could not be mistaken for a signal.

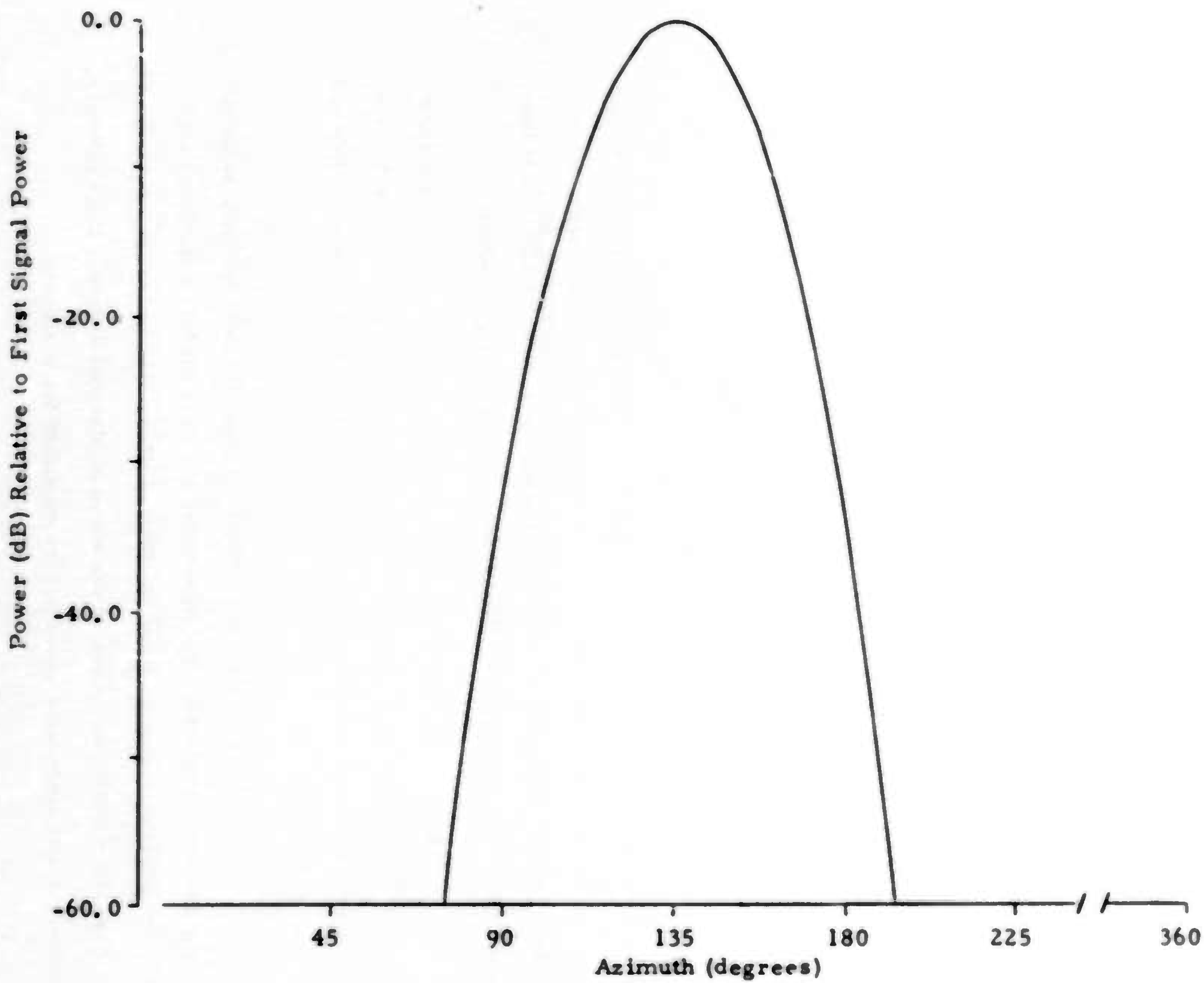


FIGURE III-2

EIGENSPECTRUM FROM FIRST EIGENVECTOR

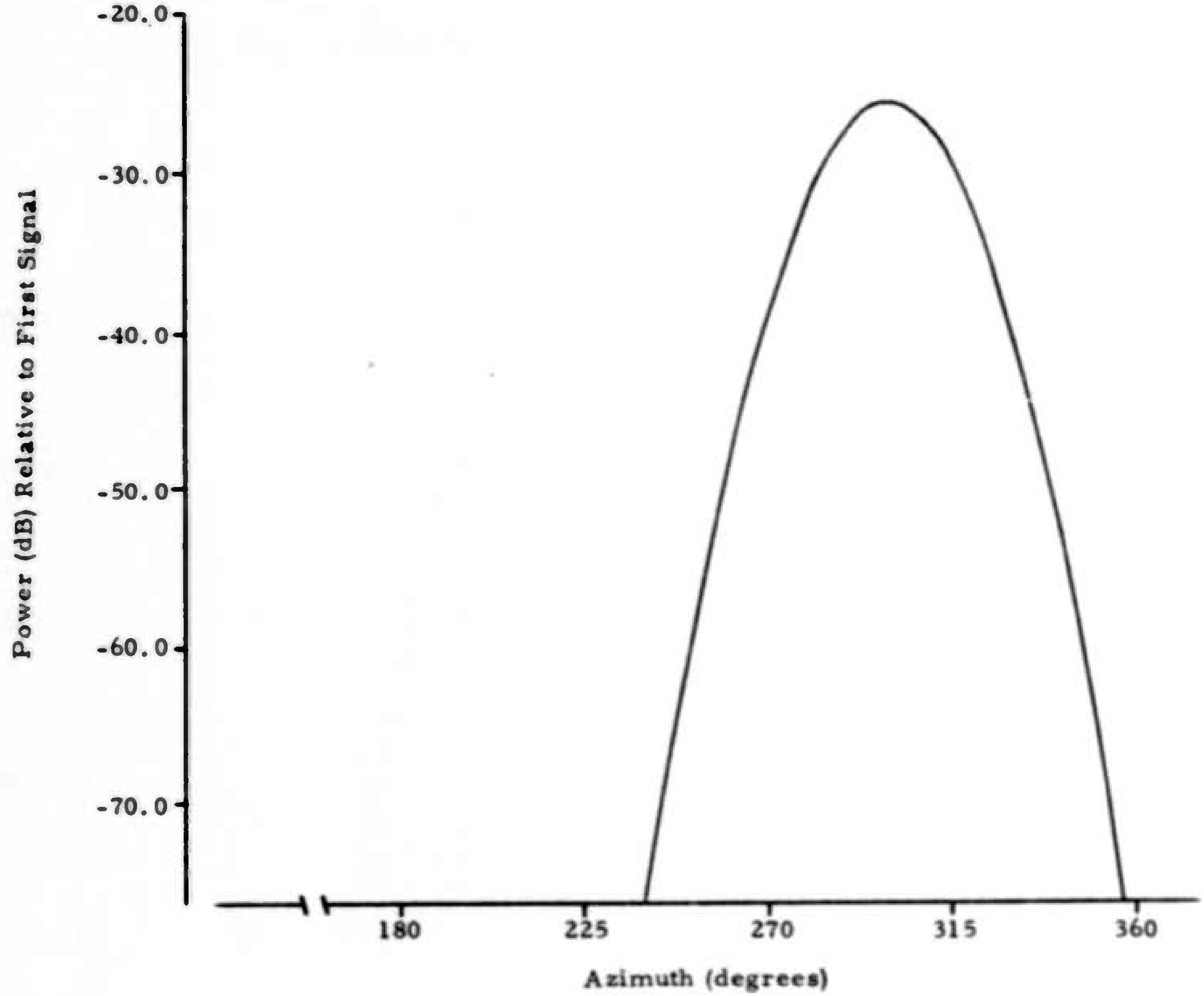


FIGURE III-3

EIGENSPECTRUM FROM SECOND EIGENVECTOR

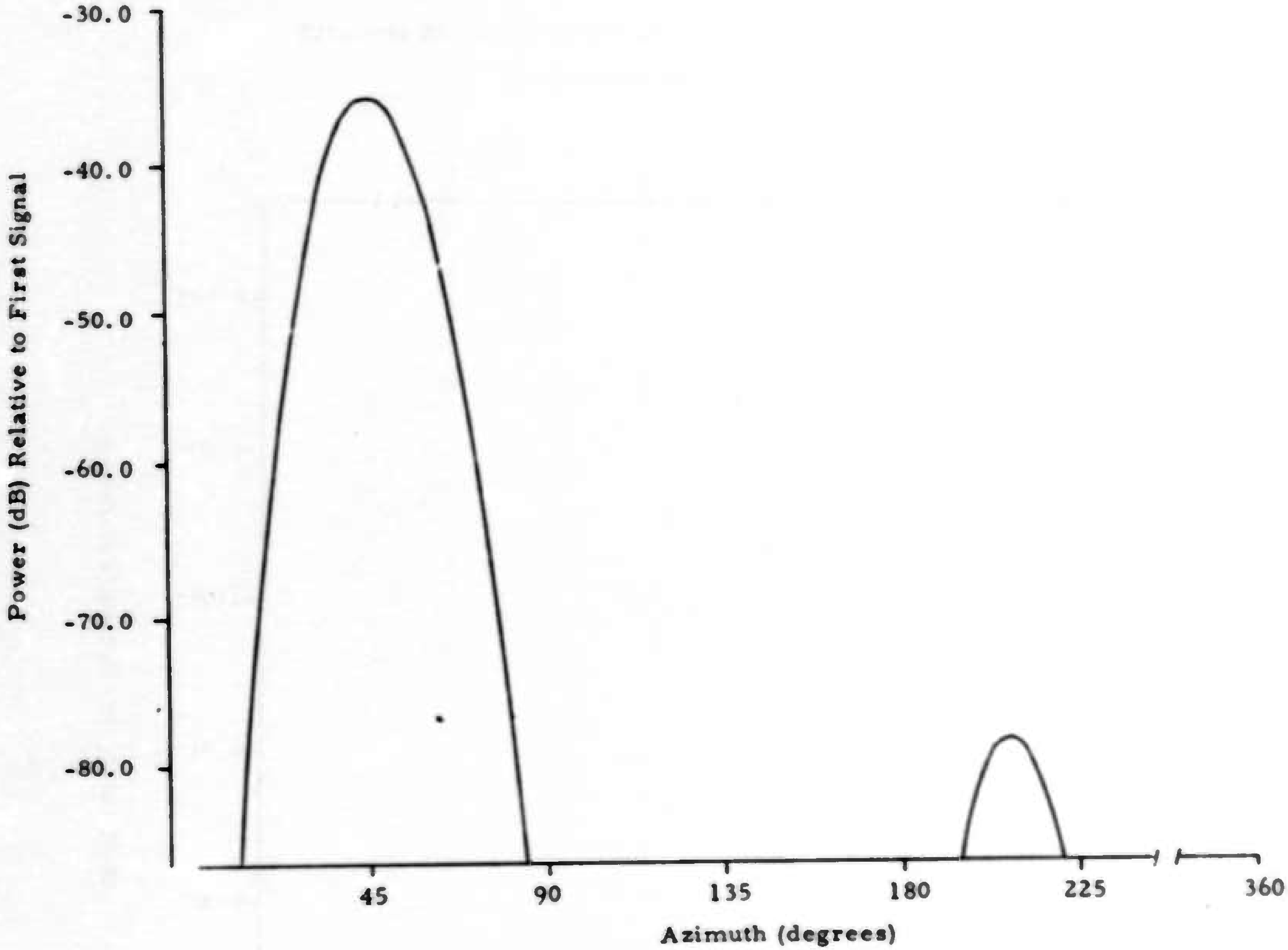


FIGURE III-4

EIGENSPECTRUM FROM THIRD EIGENVECTOR

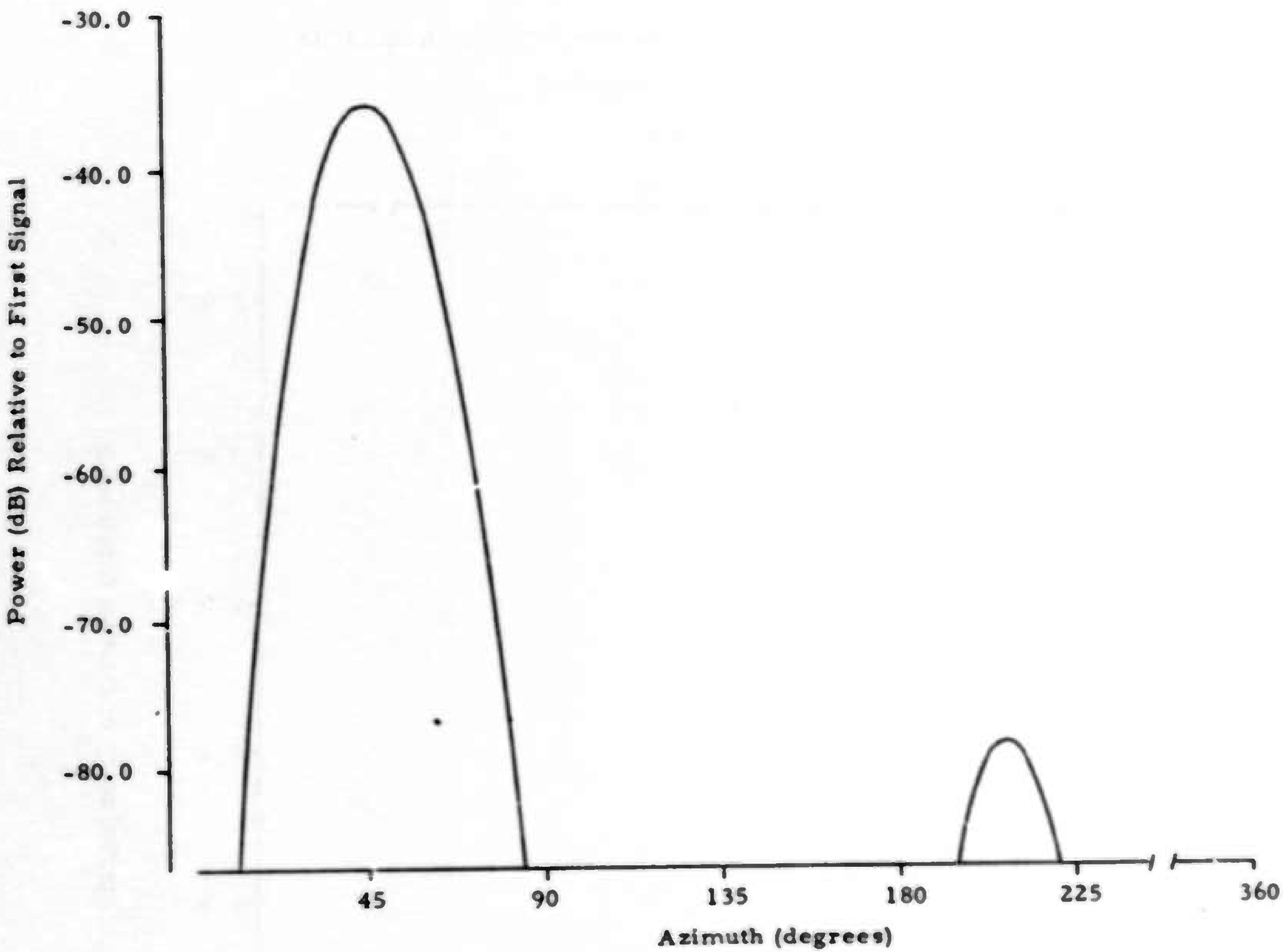


FIGURE III-4

EIGENSPECTRUM FROM THIRD EIGENVECTOR

For comparison, Figure III-5 shows the beamsteer spectrum for this signal model. Only the largest signal is resolved. The maximum-likelihood spectrum of Figure III-6 resolves both signals, but is in error by 15 dB in its estimate of the second signal's power.

From this comparison it can be seen that the eigenspectrum has substantially greater resolving power than either beamsteer or maximum-likelihood spectra for synthetic data.

## D. CONCLUSIONS

The eigenspectrum technique has the potential ability to separate interfering signals by closely estimating both their azimuth and absolute power. This ability is based on the identification of the eigenvalues of the data matrix with the signal powers, and the eigenvectors with the signal vectors. Accurate algorithms for the extraction of eigenvalues from matrices can then be used to find the signal power and azimuth.

The technique has been tested with synthetic data which simulates realistic signals in the presence of isotropic noise. Over a wide azimuthal range the eigenspectrum technique can resolve signals down to 27 dB below another interfering signal. It is superior to either beamsteering or maximum-likelihood techniques in this respect.

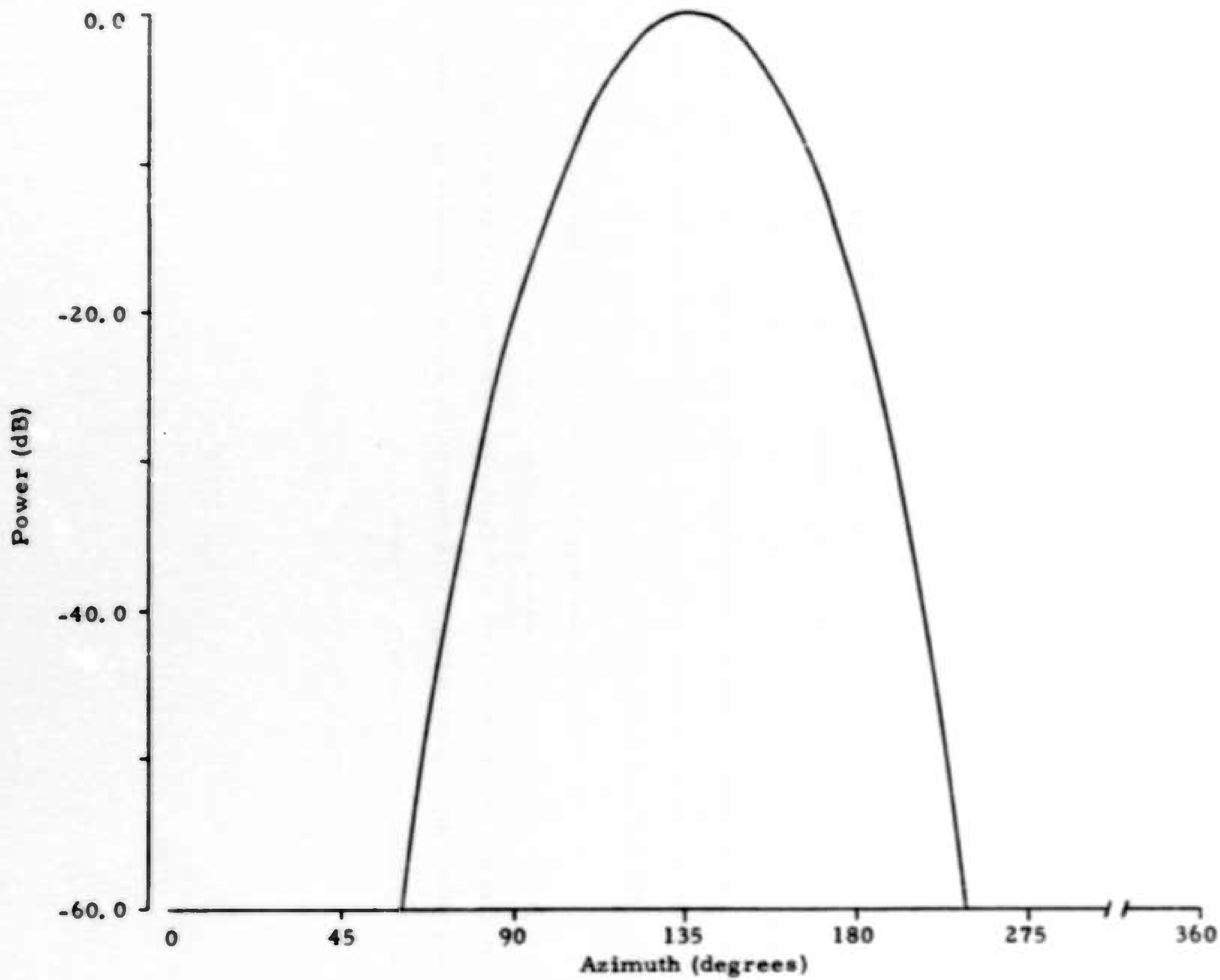


FIGURE III-5

BEAMSTEER SPECTRUM

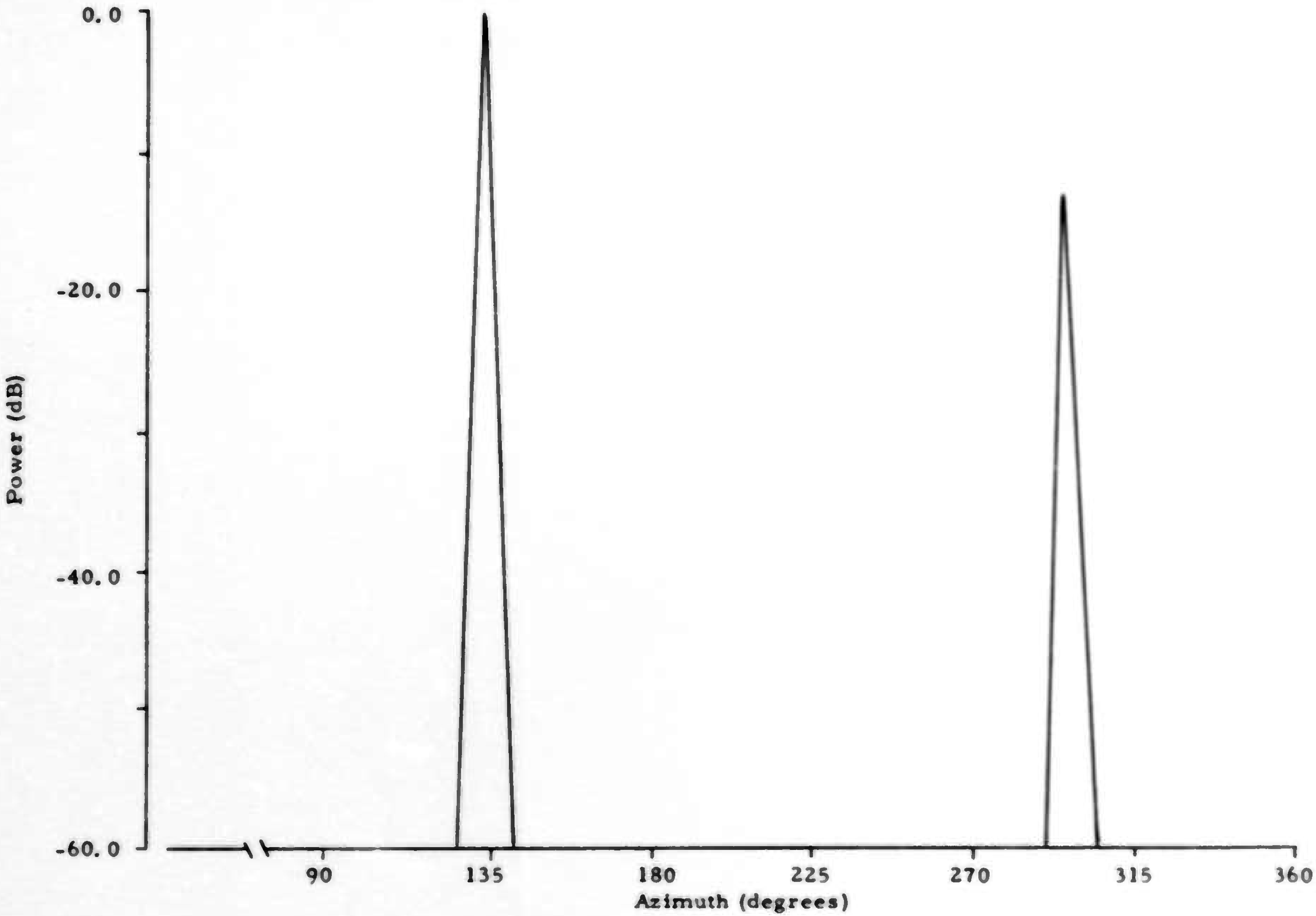


FIGURE III-6

MAXIMUM-LIKELIHOOD SPECTRUM

## SECTION IV

## CONCLUSIONS

Two counter-evasion techniques have been investigated here, using synthetic data in each case. The complex cepstrum technique is capable of separating overlapping events which are similar to one another and whose difference in arrival time is at least .7 seconds, if the second signal is not much larger than 2.0 times the first. Using experience gained in processing synthetic combinations of events, it is suggested that the event KAZ/115/04N may consist of two events, separated in arrival time by 1.1 seconds, where the second event has about one third the amplitude of the first.

The eigenspectrum technique has been shown to have marked superiority to both the beamsteer and maximum likelihood methods in its ability to find small signals in the presence of large ones, for models based on chirp waveforms. The eigenspectrum is able to extract a signal hidden in another signal of 27 dB greater strength, over a wide azimuthal range. The presence of noise did not interfere with the extraction of one signal from the other, unless the noise was as large as the smaller signal.

# SECTION V

# REFERENCES


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